Research Experiences for Undergraduates (REU) programs in mathematics are a wise national investment, an investment that is designed to produce the next generation of mathematicians. However, even though these programs were envisioned as primarily aimed at undergraduates, their impact has been much broader. They have changed mathematical culture. As just one important example of cultural change: thirty years ago it was rare for undergraduate mathematics majors to conduct research, now it is widespread, and even considered an important factor in applications to graduate schools and prestigious fellowship programs. When we look abroad we see that other countries have not made this change.

REU programs are designed to promote post-graduate study in the subject area. Participating in an REU provides much more than just a research experience for a student. Among the benefits are:

- Open-ended problems are presented, with no clear solution. As opposed to coursework, students are given the opportunity to explore, to conjecture, to formulate plans of attack.
- Computing is often a large part of an REU program and students are given the opportunity to either learn a programming language or to apply their computing skills.
- Group work is often included in REUs and these establish a network of friends and contacts, and also emphasize an important research tool, talking to each other.
- Career guidance is provided at REUs and students are given information about graduate schools.
- Faculty and graduate students who run the REU program can serve as a resource for students as they proceed along their educational path.
- The existence of REU programs serves as motivation. It shows students that hard work can lead to exciting paid travel opportunities early in their careers.

But, who can participate in these REU programs?

Mathematicians create questions or problems for students to work on in the REUs. This is quite a daunting task and mathematicians should be commended for their efforts to engage undergraduates in this fashion. The fact that students are to be engaged in research mandates that they arrive with some minimal knowledge. Most REU programs are for students who have completed the first two years of the mathematics major course of study, plus some more advanced coursework. This requirement gives the impression that third-year students would have an opportunity to apply to these REUs, that is, REUs are open to potentially all math majors at least once during their undergraduate careers.

This is far from the truth. Most mathematics majors do not select the mathematics major upon arriving at the university. Many add the mathematics major during the second or third year of
study. These students most likely will not have the above-mentioned characteristics even at the end of their third year of study, so these students do not see themselves as competitive for these REU programs. Students who attend universities with small numbers of mathematics majors also have the problem that they cannot take the required courses by the end of their junior year in order to be competitive for REUs.

How do students even know that they should apply to REU programs? Getting the information about research opportunities to students, at a time that they can utilize that information, is a challenge.

How can we provide a research experience to capable students that would serve to encourage them to pursue post-graduate studies in mathematics-based careers?

There are a few REU programs that are aimed at a different audience, namely students who are just completing their first or second year of study. If REU programs are designed to encourage students to pursue graduate studies in the mathematical sciences, then such a program is a riskier investment. Nevertheless, such programs can show students a different view of mathematics, and this alone can be motivating to students. Motivating students to the continued study of mathematics is certainly a goal that mathematicians would support. However, there are simply too many students and too few REU programs to accommodate all of these students. We would all agree that providing a research experience for undergraduate mathematics majors is a sound educational goal. It is apparent that the classroom setting will not provide such a research experience and that some structured out-of-class activity is in order. Such activities take up faculty time and this is also a barrier to providing this important experience to a large percentage of the undergraduate mathematics majors.

So, how are we to provide research experiences to first and second year mathematics majors? We recognize that these students, early in their studies, may not want to proceed onto graduate studies in the mathematical sciences, or even complete the mathematics major! Nevertheless, providing these students with a research experience so early in their studies can be transformative.

Moreover, there is another reality that we must confront. In stating that we want to encourage students to pursue advanced degrees in the mathematical sciences, we have to understand what that statement means.

**What does graduate study in the mathematical sciences mean?**

In the middle of the last century, graduate study in mathematics began with courses in abstract algebra, real/complex analysis, and topology. Mathematics departments, run by research mathematicians, structured the undergraduate program of study with a view towards graduate study and the upper division coursework of mathematics majors included courses that would prepare mathematics majors for this transition into graduate school. The upper division courses consisted of linear algebra, abstract algebra, complex variables, advanced calculus, plus other courses that represented the research interests of faculty.
Of course, there are other career paths that a mathematics major could pursue, like teaching mathematics in high school. But even for this career path, the above mentioned upper division courses played a prominent role, as the following quote shows, “Begle was dealing with high school teachers who are traditionally required to complete the equivalent of a major in mathematics. However, the requirements for math majors are designed mainly to enable them to succeed as mathematics graduate students, ….” [8].

This traditional mathematical preparation of mathematics majors is still very much a part of the culture of mathematics departments. Epsilon-delta (advanced calculus) and structure (abstract and linear algebra) are considered essential ingredients in the course work for an undergraduate mathematics major, even though graduate programs in the mathematical sciences have changed since the 1950’s. This same mathematical preparation may now be serving to discourage students from pursuing post-graduate studies in the mathematical sciences.

In the latter part of the last century there began a growth in graduate programs in applied mathematics. Many of these programs have connections to the life sciences, engineering, finance, among other areas. These are still mathematics graduate programs but their emphasis may be in applications. It is more appropriate now to talk about graduate programs in the mathematical sciences. Mathematical sciences includes traditional programs in mathematics, but is also includes applied mathematics, statistics and biostatistics. The last three do not require the traditional training of the mathematics major. Graduate programs in applied mathematics would like to see students with courses in differential equations, numerical analysis, linear algebra and advanced calculus. Graduate programs in statistics require advanced calculus, linear algebra, probability and statistics. Graduate programs in biostatistics state that they require three semesters of calculus and linear algebra. Notice that none of these programs require abstract algebra.

As a number theorist I greatly value the study of abstract algebra, but my responsibility in preparing the next generation of mathematical scientists is not to mold them to my career, but rather to prepare them to participate in the mathematical enterprise of this country.

Attracting students into the mathematics major is but the first step, retaining students in the major is an issue that must also be confronted. The structure of the undergraduate program of study is an important ingredient in the ability of a department in attracting and retaining mathematics majors. The traditional program of study for the mathematics major has one goal in mind. Prepare students for advanced study in mathematics.

The first two years of the mathematics major are fairly typical: the calculus sequence, differential equations, perhaps a lower level course in linear algebra or some course in discrete mathematics, and some course helping students to transition into “proof”. This last course is necessary as it is common for mathematics majors to take the same course in calculus as engineering students. This broad audience for calculus results in less of an emphasis on proof and students have to be provided some guidance and transition into this important tool.

There is a tremendous change in the sophistication in the mathematics major courses from the lower division courses to the upper division courses, and this change sometimes comes as a
surprise to students as they transition into abstraction. Students who performed well in calculus and differential equations, and therefore select mathematics as a major, might struggle with advanced calculus and abstract algebra. One way of supporting these students is to have options for the upper division courses.

As an example of developing options I will describe the program of study at the University of Arizona (UA).

**The program of study at The University of Arizona**

The philosophy of the mathematics major program at the University of Arizona is that by the time students complete the undergraduate degree in mathematics, students will have functioned as mathematicians, that is

- Mathematics majors have studied mathematics and have taken at least one year-long course in some topic of mathematics
- Mathematics majors have done research or applied their mathematical knowledge in some area outside of mathematics
- Mathematics majors have participated in some activity where they communicated mathematical ideas

In addition to these points, mathematics majors belong to a mathematical/scientific organization, perhaps the undergraduate math club, and they have resumes. They are professionals.

Mathematics majors at the UA are required to take at least one course in computer programming. In fact, the more programming experience a mathematics major has, the more employment opportunities there are for that student. A minor is required for all mathematics majors at the UA and these minors are an important component in finding opportunities for research and employment. My advice to mathematics majors who wish to find employment upon completion of the bachelor’s degree is that their minor should be computer science. To have the problem solving skills of a mathematician, combined with the ability to utilize the power of computing, makes for wonderful employment opportunities. The department has made it easier for students to double major in mathematics and computer science. Several mathematics courses can be used to satisfy upper-division computer science requirements in the computer science major and one course in computer science can be used to satisfy an upper-division mathematics requirement. Students who graduate with double majors in mathematics and computer science are much sought after. These majors are hired by Microsoft, Google, Lockheed-Martin, etc.

The first two years of the mathematics major at the UA are fairly typical: the three semester calculus sequence, linear algebra, differential equations, and an introduction to analysis. The above mentioned courses are taken by all mathematics majors, no matter their career track. After this core, students need to select one of seven options, which require five or six more mathematics courses. One of these options is the pre-college teaching option and students planning on becoming middle or high school mathematics teachers earn their degree from the mathematics department.
Three of the options are described below.

The Comprehensive Option is the traditional one: semester-long courses in linear algebra and complex variables and year-long courses in advanced calculus and abstract algebra.

The General/Applied Option consists of five courses: methods in applied mathematics, mathematical modeling, matrix theory, and a year-long course in either differential equations, probability/statistics or numerical analysis.

The Probability/Statistics Option consists of five courses: advanced calculus, probability theory, statistics, stochastic processes, linear algebra.

A very important point must be made about the undergraduate mathematics major program at the UA. The goal of the mathematics major program is not to produce students pursuing graduate programs in the mathematical sciences. The goal of the program is for students to take enough mathematics that they are enabled to reach the goals that they have set for themselves. Many of our students have double majors or are working on two degrees. This is very difficult to do in four years. It sometimes happens that students will change the mathematics major to a mathematics minor. This is not considered a failure. These students are pursuing other options having taken more mathematics than is required for their major.

Since 2010, each year there have been over 600 mathematics majors (more than 20% of these majors are minority students) and another 700 mathematics minors. Each year since 2010 we have graduated over 100 mathematics majors. About half of the graduates had another major. About a quarter of the graduates pursued post-graduate study, but most of the areas in post-graduate study have not been in the mathematical sciences. Many of these students selected the General/Applied Option and this extra mathematics made them more competitive for graduate programs outside of mathematics. These students were accepted into graduate programs at prestigious universities like Stanford, Harvard, MIT, Oxford, and Berkeley.

In 2011, the American Mathematical Society (AMS) recognized the Math Center in the Department of Mathematics at the UA with an award for an exemplary program in a mathematics department [5,6]. In the next section the activities of the Math Center will be described.

**The activities of the Math Center in increasing the number of mathematics majors and in increasing participation in research**

The Math Center was established more than twenty years ago. The Math Center is dedicated exclusively towards running the math major program, providing advising for students, maintaining a data base, keeping track of activities for mathematics majors, sending out weekly messages to mathematics majors on opportunities and developments, and keeping in touch in the faculty advisors. Almost all the mathematics faculty have between ten and twenty advisees.

The Math Center has a full time staff person as Coordinator of the Math Center. A faculty member, currently W. Y. Vélez, serves as Director of the Math Center.
The academic year begins with a four-jour Orientation workshop for potential or new mathematics majors on the Saturday before classes start. We stress the following points.

- Students can select one of seven options available to the mathematics major. Each option is geared towards different career paths.
- Undergraduate mathematics majors who have had study-abroad experiences, summer internships or participated in research program give brief descriptions of their experiences.
- The importance of having an up-to-date resume is presented and students are emailed a “Sample Resume” on which to construct their own.
- In an attempt to form communities, students who are taking the same course are encouraged to form study groups. Emails are exchanged among students.
- The undergraduate mathematics club presents its activities, including free tutoring for selected courses.
- Employment opportunities as Undergraduate Teaching Assistantships in the department are described.
- Research opportunities in other departments are described and the importance of taking a programming course as early as possible is stressed.

We want all of our students to apply for extra-curricular experiences, including research and teaching. As we all know grades can be a barrier. Students need to know early on that their grades will be their calling cards for years to come. But there are two other major impediments.

Every application requires one-three letters of recommendation. Where are students going to obtain the support of faculty to apply for these positions if they do not attend office hours? The Orientation workshop stresses the importance of attending office hours and establishing contact with faculty so that faculty can support the students’ goals.

The other major impediment is that students often view themselves as unqualified for research positions, even when they satisfy the minimum requirements. Students should understand that their role is to apply. It is not also to serve as evaluator of their own application. If they have the minimum qualifications, they should apply.

**Integrating mathematics majors into the scientific life of the university**

We want first-year students to apply for research positions. However, these students are for the most part not competitive for national REU programs. Since we have suggested to these students to take a programming course in their first year, students will find that there is some demand for students with computing skills. But where to find meaningful scientific employment?

Now that I have described the environment of the mathematics major program, I want to go back to the topic of integrating mathematics majors into the scientific life of the university. There is a strong tradition in the life sciences to have undergraduates work in labs. Moreover, it is common for first and second year students to be hired in these labs. Given the fact that there are increasing
applications of mathematics to the life sciences, I encourage first-year mathematics majors to take a biology or chemistry course in their first year.

Let’s look at the program of study of a typical, first-year mathematics major. By the end of the first year, these students will have taken a year of biology or chemistry, a programming course, and two semesters of calculus. If a mathematics major applies for a position in a biology lab, it is likely that the mathematics major looks better prepared than a typical biology major. Moreover, there are many mathematics majors who arrive with credit for first semester calculus, so by the end of the first year of study, these students may be in third semester calculus and/or linear algebra. Mathematically, these students look like second-year students.

But it gets better. There are dry labs on campus, that is, there are life science researchers whose research is more computational in nature or the results of their experiments are large amounts of data. Some of these researchers would prefer to have student workers whose background includes more mathematics and programming skills. Our mathematics majors are perfect for these labs!

I had a phone call from the director of a research program asking me to recommend mathematics majors for the research project. I asked what the research was about and he said that they were investigating water quality issues. I replied that it appeared to me that biology or chemistry majors would be more appropriate. The director said that this work required the use of mathematical models and it was his experience that “it was easier to teach the science to a mathematics major than the mathematics to a science major.”

My experience has been that well-prepared mathematics majors, with supporting courses in programming and science, can find meaningful research experiences outside of the mathematical enterprise. These research experiences can sometimes divert the academic goals of the mathematics major. Students may become more interested in the research area of the project than in continuing on towards advanced degrees in the mathematical sciences, or even in the mathematics major. I consider this to be a success. Our role as advisors to undergraduate students is not to force a particular path on them, but rather to help students discover for themselves where their interests and passions lie. Moreover, these students bring with them a more substantial mathematical background to the new subject area. Hopefully students will see the need to pursue more mathematical training as they proceed along their chosen academic careers. This infusion of mathematically trained students into other subject areas is only a plus.

In order to provide advice to mathematics majors about the opportunities that exist on campus for mathematics majors, the department has to be knowledgeable about their existence. Many academic departments have a long history of undergraduates working on research projects with faculty and the university most likely has a record of these activities. These opportunities are often announced on-line and on the departmental websites. National grants involving undergraduates in research are common and the university can provide information about them. Faculty in the mathematics department can be working with collaborators in other departments and this can provide further resources. The department could begin collecting information about research opportunities in other departments which can be made available to the advisors.
There is nothing more powerful than simply calling up researchers and volunteering to provide good candidates for their undergraduate research program. One could ask, “What would you like to see in a student? What background, what courses would make a student attractive to your research group?” By having this conversation with faculty in other departments the mathematician would learn better how to advise students.

One of our goals, as advisors to our students, should be integrate undergraduate mathematics majors into the scientific life of this country. If mathematics majors could have a research experience outside of mathematics in their first or second year of study, followed by research experiences in the mathematical sciences, we would produce more well-rounded graduates, some of whom might choose to pursue further mathematical studies, but with a better understanding of the way that mathematics is applied.

There are other research or internship opportunities available for students outside of the university setting. Industry and national labs have programs that support students both in summer and during the academic year. Most of these programs will require programming and mathematics majors with programming skills can be competitive. A department could make contact with local industry to see what they would like to see in a student and this could inform the department about the proposed course of study for mathematics majors. Career fairs are common at universities and the department could send a representative to the career fairs to discuss with the recruiters what they would like to see in a student.

This brings to mind the importance of having mathematics majors create resumes. Resumes are a common feature for students in engineering and business, but not so common for mathematics majors. Why? Perhaps one of the reasons for this is that the common view of mathematicians is that mathematics majors are headed towards two standard careers, pursuing graduate studies in the mathematical sciences or becoming pre-college mathematics teachers. In fact, the majority of mathematics majors do not follow either of these two career paths. Mathematics majors pursue jobs in a wide variety of business areas or pursue graduate studies outside of the mathematical sciences.

The importance of mathematical training in other academic areas has changed dramatically over the last few decades. Students who wish to pursue graduate studies in economics would be well served to have had courses in advanced calculus, probability, statistics, and linear algebra, essentially a mathematics major. Mathematical modelling is now pervasive in the life sciences and senior level courses in differential equations, linear algebra, probability and statistics make students more competitive. Even in graduate programs in engineering, the above mentioned courses increase the competitiveness of the student applicant. These facts bring a new dimension to both the question of finding opportunities for research opportunities for mathematics majors and increasing the number of mathematics majors.

**The unique role that mathematics holds in university studies—an outreach activity**

Mathematics holds a unique position in university studies. Suppose that X is some good major that a student selects. If the student adds the mathematics major to her program of study, then X and mathematics is great, no matter what X is. The study of mathematics gives the student
analytical and problem solving skills. Even if the student joins the workforce and does not do mathematics, those skills remain. Of course a person in X could argue that X also builds analytical skills and provides problem solving abilities, and this is certainly true. However, the study of mathematics provides something else. Tools! We provide the ideas and tools that are used in modeling, and the ability to use these tools is a hard-won skill, one that starts in calculus, proceeds on to a more in-depth study behind the limiting processes, then on to the study of structure. Along the way, the mathematics major is developing the ability to deal with precision and to understand when a theorem is applicable by looking at the assumptions of that theorem. In an age when many calculations are done with packaged software, a mathematics major knows when to question the appropriateness of the use of that software for a problem at hand. Yes, X and mathematics is great and students with this combination are more competitive on the job market or in graduate school.

The realization that X and mathematics is a great combination led me to develop an outreach program in my department. In March of each year, I obtain the names and email addresses of all students who have been accepted into our university. The data also includes the major that the student has selected. I have crafted letters for all of the different majors and the departmental staff sends out about 15,000 messages in March and April. We use software that takes the Excel spreadsheet, takes the name of the student and inserts the name as, Dear ***, so it appears like a personal message. The sender of the email appears to be me. So if a student replies to the email, the reply will come to me.

The tone of the letter is one of concern for the welfare of the student. For example, it contains a link to the scholarship website of the university. The letter first of all congratulates the student for having been accepted to the university. I go on to comment that the major that the student has chosen is a good one and if it is appropriate, I provide a link to where the student can look for research opportunities on campus for that major. For each letter I also provide a link to the AMS website, “Mathematical Moments”, where the link is to some description of the application of mathematics to X, the student’s chosen major. The Mathematical Moments articles are just a few paragraphs. I then go on to suggest that since mathematics is so critical to X, the student should consider adding mathematics as another major because X and mathematics make a great combination. I also provide a link to the math major website. This letter suggests that the student enroll in the highest level mathematics course that they are prepared for when they arrive at the university. They should not postpone the mathematics course necessary to their major.

I also provide a hook at the end of the letter. In a postscript I tell the students that if they plan to enroll in calculus that they should send me a message and I will reply with a copy of “Resources for calculus students”, a two page document that describes the lower division mathematics courses, provides a link to the departmental calculus webpage and gives some advice. For example, a question that I am often asked is whether or not the student should enroll in calculus II if the high school student has earned college credit for calculus I through Advanced Placement credit. This is an impossible question to answer. The advice given on this document is that if the student is thinking about doing this, then the student should go to the calculus webpage. There the student can find the final exam questions for calculus I that have been collected over the last few years. The student should work on these problems over the summer. If the student can do
most of the problems, then the student is ready for calculus II, otherwise the student should start in calculus I.

There is another good reason for placing that hook at the end of the letter. If the student replies then it gives me a chance to provide some guidance or advice to the student before the student has even arrived at the university. If the student replies then I send the Resource document and I also send along a link to some videos that we created, which I hope will motivate students to add the mathematics major. I also send along a four-year schedule of mathematics courses that shows a sample schedule for the mathematics major.

Here are some examples of replies from students.

- Thank you for contacting me! The information you have given has quite changed my perspective in what I want to major in. I will definitely be majoring in Microbiology, and I can see why a mathematics major would help my studies.
- Thank you for the helpful information. I watched the videos and they made me even more excited for college. The video about the excelling high school student who did not pass her first math test and was dropped down a level made me realize that this summer instead of sleeping and watching tv, I will take initiative and brush up on my Pre-Calc and Calc skills. Thank you for sending me those videos. They were very interesting and raised my awareness that college is definitely not like High School. College is much harder. I learned from those videos that if I continue working hard and use my resources, I will do well. Thank you for your time.
- I have always been interested in mathematics, and after looking through the online handbook, I see that there is a sensible connection between life sciences and mathematics. I am interested in learning more about the necessary steps that are required to join a degree in physiology with a degree in mathematics.
- Thank you so much for taking the time to type such a detailed response. The email is very helpful in cementing my decision. Firstly, I would love to be at the Orientation on the Saturday before classes. I think that will be a great opportunity to find out more information. Thank you for the resources on how to prepare. I want to be refreshed on the concepts I have learned already, so I will start reviewing now. I watched the videos and read the article by Sean Howe. Those gave me confidence in adding on a math minor. The benefits to more math education seem numerous, one of which, as you mentioned, would be an advantage in the workforce.

If the data that we have on the students includes the email of a parent or guardian then we copy that first message to that person. Parents can be quite impressed when a professor writes a personal note to their child. I have had several messages from parents expressing their gratitude. Parents get the feeling that someone cares about the career of their child, even in a university with 40,000 students.

Here are a couple of responses from parents.

- Thank you so much for your email and sharing with us the idea of H. considering an additional degree in mathematics. M., H. and I will certainly discuss your email. We are
looking forward to our registration trip in June. I’m sure they will have a full schedule for us but if possible, and if you are there, we will come by and introduce ourselves. It was very thoughtful of you to reach out to H.

- I wanted to take a short moment to thank you for reaching out to N and I over the last summer to introduce us to the opportunity to take Math 223 (Calculus III) as well as a dual major that included Math. N is extremely excited about what he has learned over the last semester and is truly looking forward to continuing his efforts in Math 215 (linear algebra) in the fall. Your guidance was spot on and N is excited (as am I) about his continued experiences within the math department.

I should point out that Nicholas will be graduating in May 2015 with a major in Biochemistry and minors in mathematics and computer science. Without the initial message he probably would not have considered taking so much mathematics. With the added minors he will be more competitive as he looks for post-graduate opportunities.

The messages that I send out not only increases the number of mathematics majors, but it also increases the number of mathematics majors who also have another major. Approximately one third of all mathematics majors at the UA have another major or are working on another degree in our department. It is impressive how many students are willing to work this hard to accomplish this. Since these students are double majors, these students can apply to REU programs in their other major and this greatly increases the number of opportunities available to them. In fact, we have many students who arrive with two semesters of calculus and Advanced Placement credit for some science courses. If they decide to apply to REU programs in X during their first year of study, these students actually look like second- or third- year students in X, which again makes them even more competitive.

In summary, I would like to collect together these ideas into some bullet points.

- Mathematics holds a unique place in the educational enterprise. By encouraging students to include more mathematics in their undergraduate curriculum, we are opening up opportunities for them. Having students add mathematics as a major or minor further increases the career paths that they can pursue.
- Establishing an office in the mathematics department which focusses exclusively on the mathematics majors will help a department not only manage its math major program, but also it serves as a focal point for undergraduate activities.
- Graduate programs in mathematics have morphed to graduate programs in the mathematical sciences. Preparation for these graduate programs is more diverse than it used to be and the preparation of mathematics majors should reflect this.
- Mathematics majors, with a background in programming and some science, have many internship and research opportunities, both on campus and in industry.
- The department could take an active role in locating research and internship opportunities for its majors. Making connections to other departments on campus serves as good public relations for the mathematics department.

Website and references
1. Undergraduate Math Major Website at the University of Arizona
http://math.arizona.edu/ugprogram/
2. Sample Resume and Career Assistance
http://math.arizona.edu/ugprogram/mcenter/careerassistance.html
3. Videos and other resources
http://math.arizona.edu/ugprogram/mcenter/resources
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5. AMS 2011 Award for an Exemplary Program or Achievement in a Mathematics Department, Notices of the AMS Vol. 58, #5, pp 718-721, May 2011
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