APPLIED AND COMPUTATIONAL MATHEMATICS
A NEW DEGREE FOR 21ST CENTURY DISCOVERY AND INNOVATION

Transforming Post-Secondary Education (TPSE) in Mathematics

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Problems

• Typical BS in math:
  • Not a single concept that’s less than 125 years old.
  • If students don’t supplement with statistics and/or programming classes, they are woefully ill-prepared for the workforce.
• Mathematicians historically don’t play well with others. We need interdisciplinary math programs that foster teamwork and socialization.
• Math departments have a “service department” mentality that leads to a doomed business model.
• Many mathematicians are often embarrassingly disconnected from STEM disciplines and the needs of industry. Professional development is desperately needed.
• Sustainability is difficult. Innovations often die do to burnout, retirement, and administrative assignments.
• Systemic changes will probably require an “off the shelf” solution.
Ideal Job Description for BS in Math

Qualifications:
• Able to prove basic theorems from Moore method topology without getting shot down too often
• Able to identify when a ring is a Euclidean domain
• Able to solve integrals involving partial fraction decomposition and trigonometric substitution.
• The ability to compute volumes with the shell method is a really big plus!!!
• Good row reduction skills. Can solve 3x3 linear systems.
• Able to solve power-series solutions of ODE
• Knowledge of LaTeX and Maple a big plus
Ideal Job Description for BS in Math

NONE OF THESE THINGS ACTUALLY EXIST
What do real jobs look like?
# Data Scientist Job @ Overstock.com

## Job Requirements

- Background in data-driven research.
- Strong mathematical proficiency.
- Programming experience.
- Understanding of statistical analysis methods and what they mean. Knowledge of SQL and a statistical programming language. Python or R preferred.
- Proficiency in advanced quantitative/statistical modeling and optimization techniques.
- Prior experience with business analytics, especially online retail, required. All-around talent; we want to see that you can think through a complex business problem, offer five ways to solve it, and rough up a couple of the solutions over the next couple of days.

## Skills

- Big Data Analytics
- Data Modeling, Data Mining
- Hadoop Stack, Hadoop related tools (Hive, Pig, Storm, Zookeeper, Kafka, Flume, etc)
- Spark, Scala
- Python
- R, SAS
- SQL, Aster SQL-MR
- Git
- Bayesian Statistics/Probabilistic Modeling/Programming
- Visualization
- Machine Learning and Artificial Intelligence
- Natural Language Processing
- NOSQL platforms
Basic Qualifications

- Bachelor's degree in CS, Math, Statistics or a quantitative field
- 3+ years of hands-on experience in predictive modeling and analysis
- 3+ years of hands-on experience with one of the statistical software tools: R, Python or SAS
- 3+ years of writing complex SQL queries in a high volume database environment

Preferred Qualifications

- Experience/Knowledge of machine learning techniques such as GBM, random forest etc.
- Experience in e-commerce / on-line companies in fraud / risk control functions
- Coding skills in one of the modern languages Java, Python, Scala
- Experience with visualization technologies such as Tableau
- Experience in statistical techniques such as classification, clustering, regression, statistical inference, collaborative filtering, and natural language processing, experimental design, social networking analysis, feature engineering etc.
- Compelling communication and influencing skills and participation in winning the support of management and influence the course of major strategic decisions
Data Scientist @ Intuit

Qualifications

• MS in Engineering Mathematics, Statistics, Theoretical/Computational Physics, or related field
• Solid knowledge of statistical techniques is required
• Hands-on programming experience with one or more of the following: Python, Java, R, or related languages
• 1-3+ years’ experience manipulating large datasets and using databases (e.g. SAS, R, SQL, S-Plus, etc.)
• 1-3+ years’ experience with a general-purpose programming language (e.g. C, Java, Python, etc.)
• Familiarity with basic principles of distributed computing and/or distributed databases (Hadoop, NoSQL, etc.)
• Demonstrable ability to quickly understand new concepts—all the way down to the theorems—and to come out with original solutions to mathematical issues

Responsibilities

• Having a wide-latitude in determining objectives and approaches to solution development on mission critical assignments
• Going beyond established analytical thinking and problem-solving by applying creativity to unconventional concepts and out-of-the-box solutions
• Be part of a dynamic and cohesive team of highly trained and successful researchers
• Independence in the shaping and development of projects
• Ability to quickly understand patterns within large quantity of data and to reference key characteristics using visualization techniques
• Demonstrating strong programming skills in large-scale data analysis using Python, Java, R or related software
• Leveraging strong math skills and statistical knowledge to advanced data mining and data analysis activities related to next generation Cloud technologies
If you could design a degree from the ground up, how would you do it?
# Desired Outcomes

<table>
<thead>
<tr>
<th>Want the students to</th>
<th>Want the program to</th>
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<tbody>
<tr>
<td>Become sophisticated mathematically</td>
<td>Have a strong national brand</td>
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<tr>
<td>Develop strong technical and computational skills</td>
<td>Develop tight relationships with industry</td>
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<tr>
<td>Be engaged and foster a strong work ethic</td>
<td>Seed other disciplines with exceptional graduate students</td>
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<tr>
<td>Learn to be world class thinkers and problem solvers</td>
<td>Seed mathematics with exceptional graduate students</td>
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<tr>
<td>To become leaders and work well in interdisciplinary teams</td>
<td>Help mathematics align better with industry and the needs to society</td>
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<td>To get top career and graduate program placements</td>
<td>Provide a national model that elevates mathematics in the STEM community</td>
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<td>Forge strong bonds that translate into vibrant alumni relations</td>
<td>Double as a professional development platform for faculty</td>
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<td>Learn to discern truth (!)</td>
<td>Connects faculty and practitioners to other disciplines and resources</td>
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<td>To use their powers for good (!)</td>
<td>Change the culture of mathematics</td>
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Key Program Features

• Fresh 21\textsuperscript{st} Century Content (!)
  • Design, Analysis, and Optimization of Algorithms
  • Mathematical & Statistical Modeling (Predictive Analytics)
  • High-Performance Computing
  • Big Data Technologies

• Horizontal Integration Across Multiple Quantitative Disciplines
  • Students chose a concentration in a quantitative discipline

• Soft Skills Training and Networking
  • Selling Yourself, Resumes, Interviews, Networking
  • Teamwork & Leadership, Personal & Project Management

• A Capstone Experience
  • Internships and/or Industrial Cooperation
  • Undergraduate Research
Let’s Talk More About Content!
Fresh 21st Century Content

• Design, Analysis, and Optimization of Algorithms
  • Advanced Linear Algebra
  • Advanced Calculus (Multivariable Analysis)
  • Data Structures and Graph Algorithms
  • Approximation Theory and Numerical Analysis
  • Optimization (!)

• Mathematical and Statistical Modeling
  • Distributed Computing and Big Data (MPI, Hadoop, noSQL)
  • Data Analytics (Regression, Estimation, SQL, Python)
  • Modeling with Probability and Stochastic Processes
  • Bayesian Statistics
  • Machine Learning
  • Dynamical Systems (ODE, PDE, SDE)
  • Calculus of Variations and Optimal Control
Algorithmic Thinking

“A person well trained in computer science knows how to deal with algorithms; how to construct them, manipulate them, understand them. This knowledge is preparation for much more than writing good computer programs; it is a general purpose mental tool that will be a definite aid to the understanding of other subjects, whether they be chemistry, linguistics, or music, etc. The reason for this may be understood in the following way. It has often been said that a person does not really understand something until after teaching it to somebody else. Actually a person does not really understand something after teaching it to a computer, i.e., expressing it as an algorithm…An attempt to formalize things as an algorithm leads to a much deeper understanding that if we simply try to comprehend things in a traditional way.”

—Donald Knuth

Students in our program aren’t allowed to use a package until they have coded the core algorithm themselves. They have to earn the right to use a package!
Algorithms that can and should be included in an undergraduate mathematics curriculum

- Encryption algorithms: probabilistic primes (using Fermat’s little theorem), RSA, Euclidian algorithm, Diffie-Hillman key exchange
- Solving Linear Systems and Finding Eigenvalues: Row reduction, Jacobi, Gauss-Seidel, Successive over relaxation (SOR), and Krylov methods such as Arnoldi, Lancos, GMRES
- State Estimation: Kalman, extended Kalman, particle filters, recursive least squares, etc.
- Compression: Huffman, LWZ, wavelet approximation
- Tree Search: AVL trees, Black-White trees, B-trees
- Constrained Optimization: simplex, interior point, \ell^1 regularization (which includes lasso, group lasso, compressed sensing, etc.)
- Sampling and Markov Chain Monte Carlo: Gibbs, Metropolis, Metropolis-Hastings
- Matrix Decompositions: LU, QR, SVD, etc.
- Graph Algorithms: Minimum Spanning tree, traveling salesman, breadth-first search, depth-first search
- Unconstrained Optimization: Newton, BFGS, conjugate-gradient
- Dynamic programming (backward iteration)
- Classification: Logistic regression, random forests, support vector machines, neural networks
- Multi-armed bandit problems and Markov Decision Processes
- ODE Solvers (RK, RKF, Dormand-Prince)
- PDE solvers: finite difference/element: Most of these are just linear algebra solvers
- Pseudorandom number generation: Twistor
- Splines/Interpolation: Chebyshev interpolation uses FFT, barycentric Lagrange interpolation
- Time Series: ARMA, ARIMA (these can be done with the Kalman filter, but almost nobody does it this way
Mathematical & Statistical Modeling = Scientific Method
(where your hypothesis is a mathematical relationship)

Characterization

Formulate Hypothesis

Validation

Prediction/Decision

Graphing relationships, clustering, exploring dimensionality, scaling, unsupervised learning

Optimization, differential equation, training supervised learning methods

Forward algorithms, simulations, and feedback control rules

Measuring, quantifying, and reporting the quality of the results, errors, uncertainty, etc.
50 Shades of Model Uncertainty

Black Box Models

Gray Box Models

White Box Models

Fitting Data

Combination of data and first principles

First Principles

Purely data driven

Conservation Laws

\[
\min \sum_{i=1}^{n} \ell(y_i - f(x_i))
\]

\[
u_t + \nabla \cdot f(u) = 0
\]
INTRODUCING BYU’S APPLIED AND COMPUTATIONAL MATHEMATICS PROGRAM
Background and Timeline

- BYU has a relatively pure math department with historical disdain for applied math. Several pure mathematicians consistently vote “no” in hiring and promotion of applied candidates.
- BYU’s administration has no appetite for expanding graduate studies.
- Math faculty worry about a department split.
- The Applied and Computational Math degree was originally unanimously voted down by curriculum committee (2009).
- Later department vote (2011) got 75% with the contingency that the program only be allowed if the university gives a new FTE.
- We got 3 FTEs and a half-time staff position (with heavy strings)
- Program Launched in 2013.
- NSF TUES Phase 2 grant awarded in 2013
- Today 2/3 of math majors are in the Applied and Computational Math program.
- Our major boasts the highest starting salaries of any on campus with a median salary of over 80k and several over 100k. The high this year was 150k from Apple.
- We also have great grad school placements in diverse STEM fields
Program Overview

• Freshman & Sophomore Years
  • General Education Requirements
  • Minor in Mathematics (3 Calculus, Linear Algebra, ODE, proof)
  • Intro Computer Programming (C++)
  • First Semester of Real Analysis (Abbott/Blue Rudin)

• Junior Year
  • Mathematical Analysis (Linear and Nonlinear)
  • Algorithms, Approximation & Optimization
  • Work on Concentration
  • Soft-Skills Seminar

• Summer Capstone (Internship or Research)

• Senior Year
  • Modeling w/ Uncertainty & Data
  • Modeling w/ Dynamics & Control
  • Work in Concentration

LOCKSTEP
CORE
PROGRAM

STUDENTS
ARE IN
COHORTS
# First Year Sequences

## Mathematical Analysis
- Vector Spaces
- Linear Transformations
- Inner Product Spaces
- Spectral Theory
- Metric Topology
- Differentiation
- Contraction Mappings
- Integration
- Integration on Manifolds
- Complex Analysis
- Advanced Spectral Theory
- Krylov Subspaces
- Pseudospectrum

## Algorithm Design & Optimization
- Intro Algorithms
- Graph Algorithms
- Discrete Probability
- Fourier Theory
- Wavelets
- Interpolation
- Unconstrained Optimization
- Convex Analysis
- Linear Optimization
- Nonlinear Optimization
- Dynamic Optimization
- Markov Decision Processes
First Year Labs

Mathematical Analysis

• Intro Python
• NumPy
• MatPlotLib
• Complexity/Sparse Matrices
• Linear Systems
• QR (householder)
• QR (givens)
• Markov Chain Lab
• Image Segmentation
• Facial Recognition (SVD)
• Finite Differences
• Conditioning
• Newton Cotes vs. Monte Carlo
• Sparse Grid Approximation
• Variance Reduction Methods
• Complex Analysis
• Profiling and Wrapping
• PageRank
• Arnoldi Iteration and GMRES
• The Pseudospectrum

Algorithm Design & Optimization

• Standard Library
• Object Oriented Programming
• Data Structures
• Depth/Breadth First
• Nearest Neighbor Search
• Scientific Visualization
• Maximum Likelihood Estimation
• FFT and Applications
• Wavelets
• Chebychev Polynomials
• Gaussian Quadrature
• Polynomial Interpolation
• Optimization Packages
• Line Search Methods
• Conjugate Gradient Methods
• Simplex Method
• Compressed Sensing Lab
• Interior Point Methods
• Dynamic Optimization
• Multi-Armed Bandits
Second Year Sequences

Modeling with Uncertainty & Data
- Random Spaces & Variables
- Distributions & Expectation
- Limit Theorems
- Markov Processes
- Poisson, Queuing, Renewal
- Information Theory
- Kalman Filtering & Time-Series
- Principal Components
- Clustering
- Bayesian Statistics (MCMC)
- Logistic Regression
- Random Forests
- Support Vector Machines
- Deep Neural Networks

Modeling with Dynamics & Control
- ODE Existence & Uniqueness
- Linear ODE
- Nonlinear Stability
- Boundary-Value Problems
- Hyperbolic PDE
- Parabolic PDE
- Elliptic PDE
- Calculus of Variations
- Optimal Control
- Stochastic Control
Second Year Labs

Modeling with Uncertainty & Data

- Unix Shell, Shell Scripting I-II
- Regular Expressions
- Relational Databases and SQL I-II
- Scraping with BeautifulSoup
- MPI and OpenMP
- Pandas I-IV
- Hadoop
- Spark
- JSON, XML
- Web Servers
- Ipython parallel
- MongoDB
- noSQL
- Bokeh
- MCMC I-II
- EM and HMMs
- SVM
- Random Forests
- Deep Learning

Modeling with Dynamics & Control

- Harmonic Oscillators and Resonance
- Weightloss Models
- Predator-Prey Models
- Shooting Methods and Applications
- Compartmental Models (SIR)
- Pseudospectral methods for BVP
- Lyapunov Exponents and Lorenz Attractors
- Hysteresis in population models
- Conservation Laws and Heat Flow
- Anisotropic diffusion
- Poisson equation, finite difference
- Nonlinear Waves
- Finite Volume Methods
- Finite Element Methods
- Scattering Problems
- PID Control
- LQR and LQG Control
- Guided Missiles
- Merton Model in Finance
Growing list of Concentrations

- Animation
- Biology
- Business Management
- Business Strategy
- Chemical Engineering
- Chemistry
- Civil Engineering: Geotechnical
- Civil Engineering: Structures and structural mechanics
- Civil Engineering: Transportation
- Civil Engineering: Water Resources and Environmental
- Computer Science
- Economics
- Electrical and Computer Engineering: Circuits
- Electrical and Computer Engineering: Electromagnetics
- Electrical and Computer Engineering: Signals and Systems
- Financial Markets
- Geological Sciences
- Linguistics
- Manufacturing Systems Design
- Mathematical Biology
- Mathematical Theory
- Mechanical Engineering: Dynamic Systems
- Mechanical Engineering: Fluids and Thermodynamics
- Physics
- Political Science
- Statistics
- Statistics: Actuarial Science
- Statistics: Biostatistics
Leadership and Soft-Skills Training

- Resumes
- Cover Letters
- Interviews
- Internships
- How to give a talk
- Personality Theory
- Listening
- Conflict Management
- Negotiation
- Leadership
- Running a Meeting
- Project Management
- Working in Teams
- Networking
Building the ACME Brand

There's always money in the banana stand...

...and in data science.

Visited Amazon today. The recruiters and hiring managers were really excited about the ACME program. One hiring manager that was in Data Science said "If you want industry feedback about your program, you've nailed it".

They will be coming in the fall to compete for our students. Starting salaries will be over 100k. — at Amazon Doppler.
Visiting Companies in Portland, Seattle, Bay Area, DC Area, and elsewhere

- Google
- Amazon
- Nike
- Microsoft
- Boeing
- Zillow
- MITRE
- UnitedHealth
- Milliman
- The Gap
- LinkedIn
- Apple
- Raytheon
- Mercer
- Rincon
- Sequoia
- General Dynamics
- CIA
Partial List of Internships

- Amazon
- Goldman Sachs
- Google
- Facebook
- Microsoft
- Apple
- Raytheon
- Lawrence Livermore
- FBI
- EPIC
- Rincon
- Ancestry
- NSA
- Federal Reserve (NY)
- Lincoln Labs
- Sandia NL
- PG&E
- Lucid
- Fast Enterprises
- Los Alamos
- NASA
- Intermountain Health
- UnitedHealth
- Echostar
- Bates White
- Dept Homeland Security
Progress So Far

- Size of Junior Core
  - 15 Graduated April 2015
  - 25 Graduated April 2016
  - 31 Graduated April 2017
  - 42 Will be in the Senior Core in the Fall
  - 52 are registered for the Junior Core
- Won 3 of the last 5 ACM regional coding competitions
- ACME students represent most of the Putnam team
- Excellent job and graduate school placements
Final Talking Points

- Lock-step approach is Powerful
  - Recycle, don’t review!
  - Integration across topics
  - Multi-disciplinary perspective
- Cohort Model is Effective
  - Retention
  - Socialization, Team-Building
  - Strong Alumni Base
  - They become BFFs
- Combines to make an Efficient Program
  - Costs 2 FTES (8 courses/year)
  - A few will stay for graduate school at BYU and become TAs.
- Doubles as a professional development program (!)
  - Theory is taught in the classroom
  - Computation and applications taught in the labs, which are taught by grad students
NSF TUES Phase II

- Lab Manuals
  - 96 computing labs
- Soft Skills
  - Slides
- Supporting Materials
  - Slides
- 4 Books: To be published by SIAM
  - Foundations of Applied Mathematics
    - Volume 1 Math Analysis
    - Volume 2 Algorithms, Approximation, and Optimization
    - Volume 3 Modeling with Uncertainty and Data
    - Volume 4 Modeling with Dynamics and Control
For more information see ACME.BYU.EDU